CURVE FITTING · METHOD OF LEAST SQUARES

Worked Solutions

Questions 5 – 10

Straight Line Trend — Insured Percentage

Fit a straight line and estimate the percentage of insured people in 2004.

Year	1999	2000	2001	2002	2003
% Insured	21.3	23.0	19.8	25.6	23.7

Take x = Year − 2001 (mid-year), so x: −2,−1,0,1,2 → Σx = 0

a = Σy/n = 113.4/5 = 22.68

b = Σxy/Σx² = 7.4/10 = 0.74

Fitted line: y = 22.68 + 0.74x

$$y = 22.68 + 0.74x$$

Trend Line (x = Year − 2001)

$$24.9\%$$

Estimate for 2004 (x = 3)

y = 22.68 + 0.74(3) = 24.9

Straight Line Trend — Yearly Deaths

Fit a straight line trend and estimate the death rate for 2022.

Year	2014	2015	2016	2017	2018	2019	2020
Deaths	940	912	1055	1002	977	961	888

Take x = Year − 2017 (mid-year), so x: −3…3 → $\Sigma x = 0$, $\Sigma x^2 = 28$

a = $\Sigma y/n$ = 6735/7 = 962.14

b = $\Sigma xy/\Sigma x^2$ = −136/28 = −4.857

Fitted line: y = 962.14 − 4.857x

y = 962.14 − 4.857x

Trend Line (x = Year − 2017)

≈ 938

Estimate for 2022 (x = 5)

y = 962.14 − 4.857(5) ≈ 937.86

Straight Line Trend — Cement Production

Fit a straight line trend and estimate the production in year 10.

Year	1	2	3	4	5	6	7	8	9
Prod (t)	4	5	5	6	7	8	9	8	10

Take $x' = \text{Year} - 5$ (mid-year), so x': $-4 \ldots 4 \rightarrow \Sigma x' = 0$, $\Sigma x'^2 = 60$

$a = \Sigma y/n = 62/9 = 6.889$

$b = \Sigma x'y/\Sigma x'^2 = 43/60 = 0.717$

Fitted line: $y = 6.889 + 0.717x'$

$y = 6.889 + 0.717x'$

Trend Line ($x' = \text{Year} - 5$)

≈ 10.47 t

Estimate for Year 10 ($x' = 5$)

$y = 6.889 + 0.717(5) \approx 10.47$

Fitting a Second-Degree Parabola

Fit $y = a + bx + cx^2$ to the data below.

x	0	1	2	3	4
y	1	5	10	22	38

Normal equations:

$5a + 10b + 30c = 76$

$10a + 30b + 100c = 243$

$30a + 100b + 354c = 851$

Fitted Parabola

$$y = 1.4286 + 0.2429x + 2.2143x^2$$

1.4286

a (≈ 10/7)

0.2429

b (≈ 17/70)

2.2143

c (≈ 31/14)

Straight Line Trend — Annual Production

Fit a straight line trend by the method of least squares to the production data.

Year	1973	1974	1975	1976	1977
Prod ('000)	10	12	13	10	8

Take x = Year − 1975 (mid-year), so x: −2…2 → Σx = 0, $\Sigma x^2 = 10$

a = Σy/n = 53/5 = 10.6

b = Σxy/Σx^2 = −6/10 = −0.6

Fitted line: y = 10.6 − 0.6x

y = 10.6 − 0.6x

Trend Line (x = Year − 1975)

8.8 ('000)

Estimate for 1978 (x = 3)

y = 10.6 − 0.6(3) = 8.8 · Trend is declining

Fitting an Exponential Curve y = ae^(bx)

x	1	2	3	4
y	1.65	2.70	4.50	7.35

Linearize: $Y = \ln y = \ln a + bx$ (straight line in x)

$Y = \ln y$: 0.5008, 0.9933, 1.5041, 1.9947

$n=4$, $\Sigma x=10$, $\Sigma x^2=30$, $\Sigma Y=4.9929$, $\Sigma xY=14.9785$

$b = (n\Sigma xY - \Sigma x\Sigma Y)/(n\Sigma x^2 - (\Sigma x)^2) = 9.985/20 \approx 0.4993$

$\ln a = (\Sigma Y - b\Sigma x)/n \approx 0.0001 \implies a \approx 1.000$

$a \approx 1.00$
$b \approx 0.50$

Fitted Parameters

$y \approx e^{(0.5x)}$

Fitted Curve

Check: x=1→1.65, x=2→2.72, x=3→4.48, x=4→7.39 — close match